

Drift wave stabilized by an additional streaming ion or plasma population

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It is shown that the universally unstable kinetic drift wave in an electron-ion plasma can very effectively be suppressed by adding an extra flowing ion (or plasma) population. The effect of the flow of the added ions is essential, their response is of the type $(v_{ph} - v_{f0}) \exp[-(v_{ph} - v_{f0})^2]$, where v_{f0} is the flow speed and v_{ph} phase speed parallel to the magnetic field vector. The damping is strong and it is mainly due to this ion exponential term, and this remains so for $v_{f0} < v_{ph}$.

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I. INTRODUCTION

Drift wave is called universally growing mode due to the fact that it is unstable in both fluid and kinetic descriptions, and in collisional and collisionless plasmas. The wave is self-excited and it grows due to the free energy in plasma inhomogeneity and this remains so even in plasmas with hot ions. This fact was used in our recent papers where a new paradigm was put forward for the heating of the solar corona [1, 2] and solar wind [3] by drift waves, based on stochastic heating mechanism from Refs. [4, 5]. In practical situations in lab plasmas, in order to study the mode in a controlled situation, the wave is on purpose excited and driven by an electron current [6] or by a shear flow [7]. More recent experimental studies of drift instabilities are available in Refs. [8, 9].

This is a dangerous mode in any plasma environment [10]–[15], and various effects have been studied in the past in order to stabilize it. One of them is the magnetic shear which in simple slab geometry introduces a layer, in the direction of the shear gradient, at which the mode is stabilized by resonant ions [16, 17]. So although the stabilization is kinetic by nature it is routinely described as an effect of the plasma geometry.

But in more realistic laboratory situations, the same geometry which implies the stabilization by the magnetic shear in fact includes some additional features, like toroidal mode coupling, which may completely cancel the magnetic shear stabilization [18]. Much more on these phenomena may be found in our earlier works [19–21].

Yet another way of the drift wave stabilization is by cold electrons added to the plasma. In the present work we show that this can also be done by adding flowing ions or plasma, which need not be cold at all.

II. THE MODEL AND DERIVATIONS

The geometry assumed in the derivation is such that the background magnetic field (\mathbf{B}_0) is oriented along the z -axis. We assume a static (denoted by index s) inhomogeneous and quasineutral electron-ion plasma $n_{es0}(x) = n_{is0}(x)$, penetrated by a homogeneous plasma stream (in-

dex f further in the text). We allow for the presence of electrons as well in the f -species in order to avoid the issue of excess charge in case that ions alone are added, i.e., $n_{ef0} = n_{if0} = \text{const.}$, although stabilization is mainly by the f -ions. The equilibrium density gradient is in the x -direction and the wave vector \mathbf{k} lies in the y, z -plane.

Electrostatic perturbations are assumed propagating nearly perpendicular to the magnetic field $\sim \exp(-i\omega t + i\mathbf{k}_\perp \mathbf{r} + ik_z z)$. The perturbed densities are:

$$\frac{n_{es1}}{n_{es0}} = \frac{e\phi_1}{T_{se}} \left\{ 1 + (\omega - \omega_{se}^*) \sum_{n=-\infty}^{\infty} [W(\xi_{nse}) - 1] \frac{\Lambda_n(b_{se})}{\omega + n\Omega_{se}} \right\}, \quad (1)$$

$$\frac{n_{is1}}{n_{is0}} = -\frac{e\phi_1}{T_{si}} \left\{ 1 + (\omega - \omega_{si}^*) \sum_{n=-\infty}^{\infty} [W(\xi_{n si}) - 1] \frac{\Lambda_n(b_{si})}{\omega - n\Omega_{si}} \right\}, \quad (2)$$

$$\frac{n_{ef1}}{n_{ef0}} = \frac{e\phi_1}{T_{fe}} \left\{ 1 + \tilde{\omega} \sum_{n=-\infty}^{\infty} [W(\tilde{\xi}_{fe}) - 1] \frac{\Lambda_n(b_{fe})}{\tilde{\omega} + n\Omega_{fe}} \right\}, \quad (3)$$

$$\frac{n_{if1}}{n_{if0}} = -\frac{e\phi_1}{T_{fi}} \left\{ 1 + \tilde{\omega} \sum_{n=-\infty}^{\infty} [W(\tilde{\xi}_{fi}) - 1] \frac{\Lambda_n(b_{fi})}{\tilde{\omega} - n\Omega_{fi}} \right\}. \quad (4)$$

Here,

$$W(\tilde{\xi}_{n(s,f)\alpha}) = \sqrt{2\pi} \int_{-\infty}^{\infty} \frac{x e^{-x^2/2}}{x - \tilde{\xi}_{n(s,f)\alpha}} dx,$$

and

$$\tilde{\xi}_{n(s,f)\alpha} = \frac{\tilde{\omega} - n\Omega_{(s,f)\alpha}}{k_z v_{t(s,f)\alpha}}, \quad b_{(s,f)\alpha} = \frac{k_\perp^2 v_{t(s,f)\alpha}^2}{\Omega_{(s,f)\alpha}},$$

$$\omega_{s\alpha}^* = -\frac{k_\perp v_{t\alpha}^2}{\Omega_{s\alpha} L_{ns\alpha}}, \quad \Omega_{(s,f)\alpha} = \frac{q_{(s,f)\alpha} B_0}{m_{(s,f)\alpha}},$$

α denotes the species, q_α is their charge, $n_{\alpha 0}$ is the equilibrium density, $L_{ns\alpha}$ is the inhomogeneity scale length

of static component and $\tilde{\omega} = \omega - k_z v_{\alpha 0}$ is the Doppler shifted frequency due to the streaming velocity $v_{\alpha 0}$.

We are considering the case of low plasma beta $\beta_\alpha = 2\mu_0 n_{\alpha 0} T_\alpha / B_0^2 \ll 1$ due to which the magnetic field gradient is ignored following the relation $L_{ns\alpha} / L_{Bs\alpha} \sim \beta_\alpha$ [3], where $L_{Bs\alpha}$ is the scale length of magnetic field inhomogeneity. The parallel integration gives rise to the plasma dispersion function $W(\xi_{n(s,f)\alpha})$ with the argument $\xi_{n(s,f)\alpha}$, where the perpendicular integration yields the modified Bessel function in the term $\Lambda_n(b_{(s,f)\alpha}) = e^{-b_{(s,f)\alpha}} I_n(b_{(s,f)\alpha})$ with the argument $b_{(s,f)\alpha}$. For the static component $v_{s\alpha 0} = 0$, $\tilde{\omega} = \omega$, and $\omega_{se}^*, \omega_{si}^* \neq 0$, while for streaming particles $v_{f\alpha 0} \neq 0$ and $\tilde{\omega} = \omega - k_z v_{f\alpha 0}$, $\omega_{f\alpha}^* = 0$. The drift frequencies for electrons and ions are related as $\omega_{si}^* = -(T_{si}/T_{se}) \omega_{se}^*$ where $\omega_{se}^* > 0$.

The dispersion relation for the electrostatic drift waves is obtained from linearized Poisson's equation

$$\varepsilon_0 k^2 \phi_1 = -e(n_{es1} + n_{ef1} - n_{is1} - n_{if1}).$$

The Larmor radii of electrons in both plasmas are very small as compared to the ions, which allows for the expansion of the modified Bessel function for small argument as $\Lambda_n(b_{(s,f)e}) = [b_{(s,f)e}/2]^n / n!$. It is easy to see that only $n = 0$ terms survive in the limit of a negligible value of the argument, i.e., for $b_{(s,f)e} \rightarrow 0$, $\Lambda_0(b_{(s,f)e}) = 1$. We shall also separate the $n = 0$ terms in the ions contribution.

Using the identity $\Lambda_n(x) = \Lambda_{-n}(x)$ and the expansion of the plasma dispersion function for $n \neq 0$ terms in limit of the large argument, and assuming the realistic low frequency case for both the components, i.e., $\xi_{nfi}, \xi_{nsi} \gg 1$ and $\omega \ll \Omega_{(s,f)i}$, respectively, one can easily prove that the $n \neq 0$ terms vanish from the last terms of Eqs. (1-4) and we get the dispersion relation

$$\varepsilon \equiv 1 + \frac{1}{k^2 \lambda_{Dse}^2} [A_s + A_f] = 0, \quad (5)$$

where

$$A_s = \left\{ 1 + \frac{n_{is0} T_{se}}{n_{es0} T_{si}} + \left(1 - \frac{\omega_{se}^*}{\omega} \right) (W(\xi_{0se}) - 1) + \frac{n_{is0} T_{se}}{n_{es0} T_{si}} \left(1 - \frac{\omega_{si}^*}{\omega} \right) [\Lambda_0(b_{si}) (W(\xi_{0si}) - 1)] \right\}, \quad (6)$$

$$A_f = \frac{n_{if0} T_{se}}{n_{es0} T_{fe}} \left\{ \frac{T_{fe}}{T_{fi}} \left[1 + \Lambda_0(b_{fi}) (W(\xi_{0fi}) - 1) \right] + W(\xi_{0fe}) \right\}. \quad (7)$$

In order to calculate the growth rate of the drift wave, we separate the real and the imaginary parts of Eq. (5) and the growth rate becomes

$$\gamma = -\frac{\varepsilon_i}{\partial \varepsilon_r / \partial \omega_r} = -\frac{\text{Im}[A_s + A_f]}{\partial (\text{Re}[A_s + A_f]) / \partial \omega_r}. \quad (8)$$

The real dispersion relation may be obtained by taking $\varepsilon_r = 0$, i.e.,

$$1 + \text{Re}[A_s + A_f] / (k^2 \lambda_{Dse}^2) = 0.$$

The wave behavior will be discussed in two different frequency limits.

A. Specific frequency limits

a. In what follows the electrons and ions in the static component satisfy the following frequency limits $k_z v_{tsi} \ll \omega \ll k_z v_{tse}$, while for the streaming species we have

$$k_z v_{tfe} \ll |\omega - k_z v_{f0}| \ll k_z v_{tfe}. \quad (9)$$

The growth rate for the drift wave becomes

$$\begin{aligned} \gamma_1 = -c_1(g_1 + g_2 + g_3) &\equiv -\sqrt{\frac{\pi}{2}} \frac{\omega_r^2 n_{es0} / n_{is0}}{\omega_{se}^* \Lambda_0(b_{si})} \\ &\times \left\{ \left[\left(\frac{\omega_r - \omega_{se}^*}{k_z v_{tse}} \right) \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{tse}^2} \right) \right. \right. \\ &+ \Lambda_0(b_{si}) \frac{n_{is0} T_{se}}{n_{es0} T_{si}} \frac{\omega_r - \omega_{si}^*}{k_z v_{tsi}} \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{tsi}^2} \right) \Big] \\ &+ \frac{n_{if0} T_{se}}{n_{es0} T_{fe}} \frac{\omega_r - k_z v_{f0}}{k_z v_{tfe}} \left[\Lambda_0(b_{fi}) \frac{T_{fe}}{T_{fi}} \exp \left(-\frac{(\omega_r - k_z v_{f0})^2}{2k_z^2 v_{tfe}^2} \right) \right. \\ &\left. \left. + \sqrt{\frac{T_{fi} m_e}{T_{fe} m_i}} \exp \left(-\frac{(\omega_r - k_z v_{f0})^2}{2k_z^2 v_{tfe}^2} \right) \right] \right\}. \quad (10) \end{aligned}$$

Here, the meaning of the terms g_1, g_2, g_3 is obvious, and they describe contribution of static electrons and ions, and flowing electrons and ions, respectively. The real part of the frequency may be written as

$$\omega = \frac{\omega_{se}^* \Lambda_0(b_{si})}{\frac{n_{es0}}{n_{is0}} + k_\perp^2 \rho_{ss}^2 + \frac{n_{if0}}{n_{is0}} \frac{T_{se}}{T_{fe}} \left(1 + k_\perp^2 \rho_{sf}^2 \right)}. \quad (11)$$

Here, $\rho_{s(s,f)} = c_{s(s,f)} / \Omega_i$, $c_{s(s,f)}^2 = T_{(s,f)e} / m_i$. The static ions term g_2 in Eq. (10) cause damping regardless of parameters, while the electrons term g_1 yields the usual kinetic instability provided that necessary condition $\omega_r < \omega_{se}^*$ is satisfied.

As for the contribution of the flowing plasma, the g_3 -term, it turns out that the universally growing mode can completely be stabilized and this will be demonstrated below using some parameters that may be applicable to the laboratory plasma conditions.

We choose parameters which will show that damping by f -plasma is essentially due to their flow. We take $B_0 = 2$ T, $L_{nsi} = L_{nse} = L_n = 0.1$ m, $T_{si} = T_{se} = T_s = 10^5$ K, $n_{is0} = n_{es0} = n_{s0} = 10^{19}$ m $^{-3}$, $\lambda_\perp \equiv \lambda_y = 3$ mm,

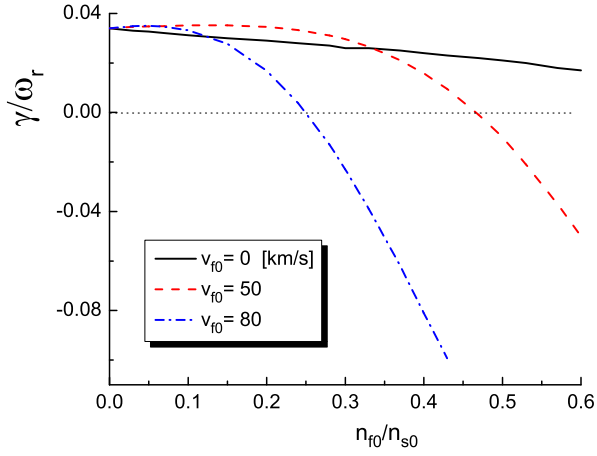


FIG. 1: The imaginary part of the drift wave frequency (10) normalized to ω_r in terms of the flowing plasma density, for the same temperature of both plasmas.

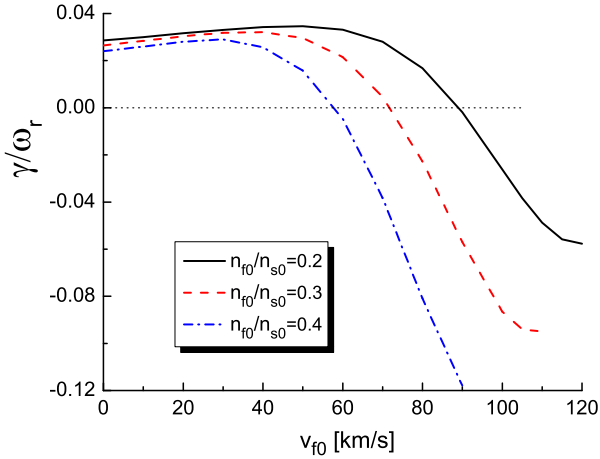


FIG. 2: The imaginary part of the drift wave frequency (10) normalized to ω_r in terms of the f -plasma speed, for the same temperature of both plasmas.

and take $k_z/k_y = 0.0002$. For such parameters the drift wave is unstable, $\omega_r = 74659$ Hz, $\gamma_1/\omega_r = 0.034$ in spite of so hot s -ions. When f -plasma particles are added, and with the same temperature $T_{fi} = T_{fe} = T_s$, there is very little change in γ_1 and the wave remains growing even if n_{f0} is strongly increased, and this remains so as long as f -particles do not flow (see full line in Fig. 1).

But if the f -plasma is flowing, the growth/damping is changed. In this case the initial instability caused by the electron term g_1 is first increased for small n_{f0} , but for larger f -plasma density the mode is heavily damped. The effect of the flow of the added plasma is thus essential, it has a profound effect on the drift wave. See more details in Fig. 2 which shows that it is possible to find particular speed values for which the mode is most effectively damped. Note that here $v_{f0} < v_{ph} \equiv \omega_r/k_z = 148530, 137104, 127311$ km/s for $n_{f0}/n_{s0} = 0.2, 0.3, 0.4$.

The effect of the flow may be understood from Fig. 3, where the f -plasma term g_3 is presented in terms of v_{f0}

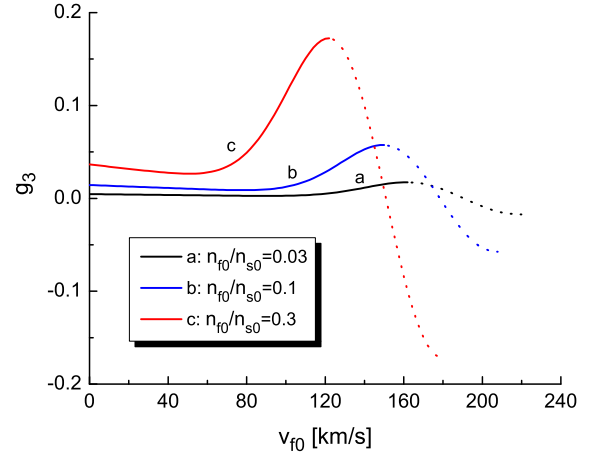


FIG. 3: The flowing plasma term g_3 from Eq. (10) used in Figs. 1, 2, in terms of the f -plasma speed v_{f0} . The peaks are the f -ion exponential term contribution.

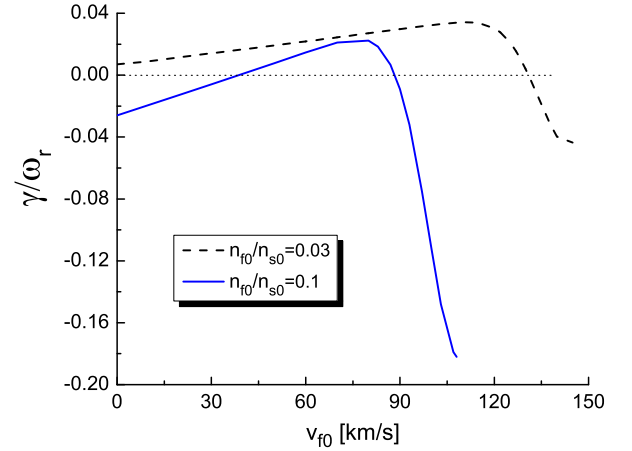


FIG. 4: The growth rate/damping (10) for a cooler f -plasma $T_f/T_s = 1/5$ in terms of the speed v_{f0} .

for the three values n_{f0} . The essential part is the ion term, which is of the shape $(v_{ph} - v_{f0}) \exp[-(v_{ph} - v_{f0})^2]$, so that normally destabilizing first part $v_{ph} - v_{f0}$ is counteracted by the ion exponential part, and the f -ion part in g_3 goes to zero for large v_{f0} instead of linearly increasing the growth rate indefinitely due to $(v_{ph} - v_{f0})$ term alone (for $v_{f0} > v_{ph}$). The electrons have a minor role and contribute only in the range $v_{f0} > v_{ph}$ when the flow destabilizes the mode (not presented here). The lines are made broken to mark regions where $k_z v_{t fi} \ll |\omega - k_z v_{f0}|$ is violated and this analytical model should not be used, hence the given speed limit here and in Figs. 4, 6.

To check the effects of the f -plasma temperature, in Fig. 4 we give the imaginary part of frequency in terms of v_{f0} for the two densities n_{f0}/n_{s0} and for $T_f/T_s = 0.2$. It is seen that for $n_{f0}/n_{s0} = 0.1$ the mode is immediately damped even for $v_{f0} = 0$ as soon as the f -plasma is added, and there is a strong damping for larger v_{f0} . The dependence of γ_1 on T_f is complicated, but Fig. 4 may partly be understood from Fig. 5 where we set $n_{f0}/n_{s0} =$

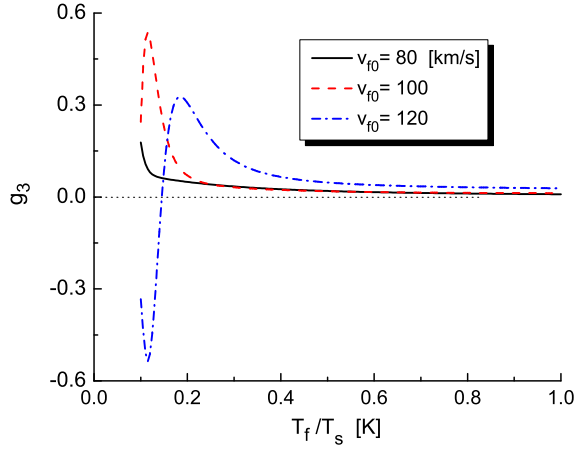


FIG. 5: Flowing plasma term g_3 in the imaginary part of frequency (10) in terms of f -plasma temperature.

0.1; it is seen that in the given temperature range, around $T_f/T_s = 0.2$, the g_3 term in (10) is positive and it causes strong damping, but this is not always so.

b. In the frequency range

$$|\omega - k_z v_{f0}| \ll k_z v_{t fi}, \quad (12)$$

the f -ion response is nearly Boltzmannian while electron contribution is negligible (see further in the text), and with similar approximations as above we have:

$$\begin{aligned} \gamma_2 &= -c_2(\alpha_1 + \alpha_2 + \alpha_3) \\ &= -\sqrt{\frac{\pi}{2}} \frac{\omega_r^2}{\omega_{se}^* \Lambda_0(b_{si})} \left[\left(\frac{\omega_r - \omega_{se}^*}{k_z v_{t se}} \right) \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{t se}^2} \right) \right. \\ &\quad \left. + \frac{T_{se}}{T_{si}} \Lambda_0(b_{si}) \left(\frac{\omega_r - \omega_{si}^*}{k_z v_{t si}} \right) \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{t si}^2} \right) \right. \\ &\quad \left. + \frac{n_{f0} T_{se}}{n_{s0} T_{fi}} \Lambda_0(b_{fi}) \frac{\omega_r - k_z v_{f0}}{k_z v_{t fi}} \right], \quad (13) \end{aligned}$$

$$\omega_r \approx \frac{\omega_{se}^*}{1 + k_\perp^2 \rho_{ss}^2 + \frac{n_{f0} T_{se}}{n_{s0} T_{fi}}}. \quad (14)$$

Here, $n_{es0} = n_{es0} = n_{s0}$, $n_{ef0} = n_{if0} = n_{f0}$. The f -ion term α_3 causes a strong damping when v_{f0} is small and this can be checked for the same parameters as before. However, for $v_{f0} > v_{ph}$ the wave is destabilized and this can easily be more efficient than in the case of electron-current driven mode [1, 6]. Indeed, in the usual electron-ion plasmas, the latter implies an additional electron current term $\alpha_4 = u_0/v_{Te}$ in the growth rate (13), but this can easily be smaller than the existing α_3 term. For $v_{f0} > v_{ph}$ we have that $\alpha_3 > u_0/v_{Te}$ if $v_{f0}/u_0 > (v_{t fi}/v_{Te})(T_{fi}/T_{se})(n_{s0}/n_{f0})$. Taking v_{Te} as

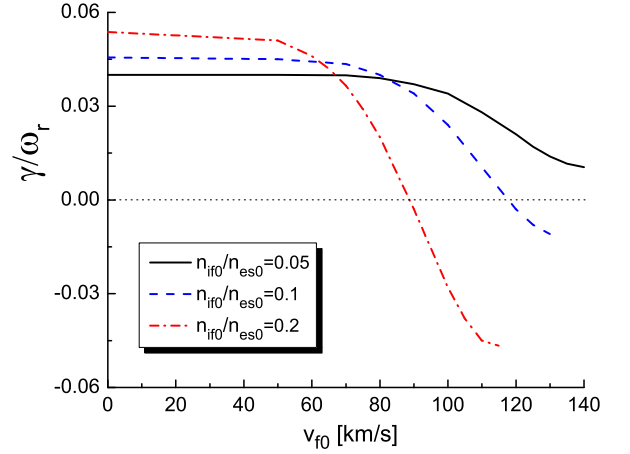


FIG. 6: Drift wave stabilized by flowing ions.

our $v_{t se}$, here the right-hand side can clearly be much below unity, so the ion flow in this regime can be far more efficient in *exciting* the drift mode.

The omitted electron terms make only minor changes in Eqs. (13, 14): α_3 term is multiplied by a small factor $1 + (m_e/m_i)^{1/2}(T_{fi}/T_{fe})^{3/2}$, and the last term in denominator of Eq. (14) is multiplied by a term $1 + T_{fi}/T_{fe}$.

B. Flowing ions case

We checked the case of adding flowing ions only, in the range $k_z v_{t fi} \ll |\omega - k_z v_{f0}|$, assuming that plasma adjusts in such a way that global quasineutrality is preserved $n_{es0} = n_{is0} + n_{if0}$. This is completely equivalent to Ref. [22] where the stabilization is discussed by an additional cold *electron* population. In Eqs. (10, 11) vanish the f -electron term, and the factor 1, respectively. The result is presented in Fig. 6 for several densities of the flowing ions and the result is similar to Fig. 2. The frequency is $\omega_r = 70926, 67193, 59723$ Hz for $n_{fi0}/n_{se0} = 0.05, 0.1, 0.2$. Here we keep $T_{fi} = T_{si} = T_{se} = 10^5$ K, and other parameters are the same as before. The wave behavior is very similar to the previous plasma flow case.

Here, the f -ion flow in principle implies a current that might cause a sheared magnetic field component $B_s = \mu_0 e n_{f0} v_{f0} L_s$, where L_s is the characteristic shear length, which is known to stabilize the drift wave itself [17, 23]. However, for parameters used in the text the sheared component is negligible; at the perpendicular distance $L_s = L_n$ it remains below $0.001 B_0$. At shorter distances it is even smaller and can be neglected.

III. SUMMARY

In conclusion, this work provides some clear recipes for damping of the drift wave which is usually believed to be universally unstable. The stabilization is expected to work for various modes from the drift wave spectrum

and it can be used as an alternative for some other mechanisms proposed in the past [24–26].

The model presented here also has an obvious advantage with respect to so called stabilization by cold electrons (having some temperature T_c) [22] because the latter disregards simultaneous cold electron collisions with other species (which is proportional to $1/T_c^{3/2}$, so the cooler electrons the more collisions). Hence, these collisions can be frequent even if the plasma is fairly collisionless regarding its usual components (ions and hot electrons). On the other hand, thermalization of such cold

electrons is instant and its characteristic time is the same as their velocity relaxation, and they are thus totally inefficient in stabilizing the drift mode. So the ion (plasma) flow stabilization presented here is clearly a far better alternative.

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